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Efficient microwave-induced optical frequency conversion

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Abstract. Frequency conversion process is studied in a medium of atoms with a Λ configuration of levels, where transition between two lower states is driven by a microwave field. In this system, conversion efficiency can be very high by virtue of the effect of electromagnetically induced transparency (EIT). Depending on intensity of the microwave field, two regimes of EIT are realized: "dark-state" EIT for the weak field, and Autler-Townes-type EIT for the strong one. We study both cases *via* analytical and numerical solution and find optimum conditions for the conversion.

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1 Introduction

Frequency conversion is a useful technique for generation of coherent tunable radiation [1]. Efficient conversion of a continuous-wave radiation at relatively low pump intensities requires high nonlinear optical susceptibility of an (atomic) medium, which can be achieved by tuning to resonances. However, this will also increase the medium absorption and refraction, seriously limiting the conversion efficiency. It was recently proposed [2] and demonstrated in many experiments that this problem can be overcome if one uses the effect of electromagnetically induced transparency (EIT) [3]. For example, the UV radiation has been generated by use of DC electric-field coupling in atomic hydrogen [4]. Radiation fields have been used to produce transparency in experiments on red to blue frequency conversion with molecular sodium [5], and on enhanced four-wave mixing with doped crystals [6]. Recently, blue to UV [7] and UV to VUV [8] conversion in atomic Pb vapor have been reached with almost unity photon-conversion efficiency.

EIT is due to quantum interference in multilevel quantum systems (atoms, molecules, dopants in solids) induced by applied electromagnetic radiation. There are two basic mechanisms responsible for EIT. In the first one, the cancellation of absorption and refraction can be explained by creation of a coherent superposition of atomic states ("dark" state) not excited by the radiation, and by preparation of atoms in this superposition (which is termed coherent population trapping – CPT) [9]. The dark state has to be radiatively stable in order to allow for the population trapping. Therefore, the dark state must be a superposition of the metastable atomic states. The second mechanism relies on a large strength of one, "coupling", electromagnetic field which mixes and splits the states (Autler-Townes effect [10]). When another, weaker "probe" field is tuned in between the two mixed states, it experiences reduced absorption not only because of the splitting but also due to interference between excitation paths to two mixed states. This mechanism works well even in the case when the states mixed by the coupling field decay spontaneously.

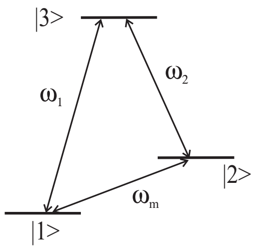

Fig. 1. Λ system with two metastable states $|1\rangle$ and $|2\rangle$. ω_1 and ω_2 are the optical frequencies, $\omega_{\rm m}$ is the microwave frequency.

In the present paper we consider the frequency conversion in a scheme where both mechanisms of EIT are possible. This is a three-level Λ system (Fig. 1), where $|1\rangle$–$|3\rangle$ and $|2\rangle$–$|3\rangle$ are the electric-dipole optical transitions, and the microwave (m.w.) transition $|1\rangle$–$|2\rangle$ is a magnetic-dipole one. Such systems can be realized, *e.g.*, on D-lines in alkali atoms, and may also be found in some molecules and doped crystals as well [11–13].

When one of the optical fields (let say, ω_1) and the m.w. field $\omega_{\rm m}$ are applied to the Λ atom, they induce an optical polarization on transition $|2\rangle$–$|3\rangle$. This leads to the generation of the optical field with frequency ω_2. In general, this field as well as the field ω_1 will be fast absorbed, if they are tuned close to the resonance. However,

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the absorption can be substantially reduced for particular values of the microwave intensity. For the weak m.w. field, optical waves create the dark state which is only slightly disturbed. The strong m.w. field mixes and splits ground states $|1\rangle$ and $|2\rangle$ so that relatively weak optical fields experience EIT of the Autler-Townes type (as we show below, in this case EIT is only due to splitting, not due to interference). In the present paper we consider both cases, treating an interaction of the e.m. radiation with atoms as well as the propagation of radiation through the medium in exact manner.

2 Model and general results

The optical waves propagation along the z-axis in the medium is described by the Maxwell equations. In the slowly varying amplitude and phase approximation, and in continuous-wave limit these equations can be reduced to the following form [14,15]:

$$\frac{\mathrm{d}g_n}{\mathrm{d}\zeta} = -\mathrm{Im}(\tilde{\sigma}_{3n}), \tag{1a}$$

$$\frac{\mathrm{d}\varphi_n}{\mathrm{d}\zeta} = -\frac{1}{g_n}\mathrm{Re}(\tilde{\sigma}_{3n}), \tag{1b}$$

where $g_n = d_n E_n/2\hbar\gamma_1$ are the dimensionless optical field amplitudes (Rabi frequencies), E_n , φ_n and $\mathbf{e}_n$ $(n = 1, 2)$ are the field amplitudes, phases and unit polarization vectors, respectively; γ_1 is the spontaneous decay rate in a channel $|3\rangle \to |1\rangle$, and $d_n = \langle 3|\mathbf{e}_n\cdot\hat{\mathbf{d}}|n\rangle$ are the matrix elements of the electric-dipole moment operator $\hat{\mathbf{d}}$ in the basis of atomic states $|l\rangle$, $l = 1, 2, 3$. The dimensionless optical length ζ is expressed in terms of an absorption cross-section for the optical field

$$\zeta = \left(2\pi\omega_1 d_1^2/c\hbar\gamma_1\right) Nz = \left(3\pi c^2/2\omega_1^2\right) Nz,$$

N is the density of active atoms.

The medium optical polarization components (the right-hand side of Eqs. (1)) are determined by the (steady-state) density matrix elements σ_{3n} averaged over the atomic velocities with the distribution $w\left(v_z\right)$, where v_z is the z-projection of the atom velocity:

$$\tilde{\sigma}_{3n} = \int_{-\infty}^{+\infty} \mathrm{d}v_z w\left(v_z\right)\sigma_{3n}\left(v_z\right),$$

with

$$\sigma_{3n}\left(v_z\right) = \rho_{3n}\left(v_z\right)\exp\left[\mathrm{i}\left(\omega_n t - k_n z + \chi_n\right)\right],$$

where $\rho_{3n} \equiv \langle 3|\hat{\rho}|n\rangle$, $\hat{\rho}$ is the atomic density matrix. The phase χ_n is the sum of the e.m. field phase φ_n and the phase ϑ_n of the atomic dipole moment $d_n = |d_n|\,\mathrm{e}^{\mathrm{i}\vartheta_n}$: $\chi_n = \varphi_n + \vartheta_n$. Similar quantities are determined for the microwave transition: Rabi frequency $g_\mathrm{m} = \mu H/2\hbar\gamma_1$ with the m.w. field amplitude H and phase φ_m; matrix element $\mu \equiv \langle 1|\hat{\mu}|2\rangle = |\mu|\,\mathrm{e}^{\mathrm{i}\vartheta_\mathrm{m}}$ of the magnetic-dipole moment $\hat{\mu}$, and the m.w. transition phase $\chi_\mathrm{m} = \varphi_\mathrm{m} + \vartheta_\mathrm{m}$.

Presence of the field on $|1\rangle$–$|2\rangle$ transition and/or both optical fields means, corresponding to the Maxwell equations, that the m.w. wave should also change along the propagation path. The EIT-assisted generation of a microwave radiation has recently been observed in atomic Cs vapor [16]. We, however, will not consider this effect here since the changes are of the order of $\Delta g_\mathrm{m}^2 \approx \left(\omega_\mathrm{m}/\omega_1\right) g_1^2 \approx \left(10^{-8} \div 10^{-5}\right) g_1^2$ at the most [17] which is negligible in the present context. Moreover, the propagation direction of the m.w. wave (traveling or standing one in a m.w. cavity) can be chosen perpendicular to the z-axis.

Let us now consider the case when all three e.m. fields are in resonance with corresponding transitions. This situation can be studied analytically if we additionally suppose equal spontaneous relaxation rates $\gamma_1 = \gamma_2 \equiv \gamma$, zero relaxation rate of the coherence between states $|1\rangle$ and $|2\rangle$: $\varGamma = 0$, and zero atomic velocity $v_z = 0$. Solution of the density-matrix equations for this case is given in the earlier work of one of us [14]. Nevertheless, we display it here again since it is important for further consideration:

$$\mathrm{Im}(\sigma_{31}) = -\frac{g_2 g_\mathrm{m} g_0^2\left(g_0^2 - 2g_\mathrm{m}^2\right)}{2L}\sin\varPhi + \frac{g_\mathrm{m}^2 g_1\left(g_1^2 - g_2^2 + 2g_2^2\sin^2\varPhi\right)}{L}, \tag{2a}$$

$$\mathrm{Im}(\sigma_{32}) = \frac{g_1 g_\mathrm{m} g_0^2\left(g_0^2 - 2g_\mathrm{m}^2\right)}{2L}\sin\varPhi - \frac{g_\mathrm{m}^2 g_2\left(g_1^2 - g_2^2 - 2g_1^2\sin^2\varPhi\right)}{L}, \tag{2b}$$

$$\mathrm{Re}(\sigma_{31}) = \frac{g_2 g_\mathrm{m}\left(g_1^2 - g_2^2\right)\left(g_0^2 - 2g_\mathrm{m}^2\right)}{2L}\cos\varPhi + \frac{g_\mathrm{m}^2 g_1 g_2^2}{L}\sin 2\varPhi, \tag{3a}$$

$$\mathrm{Re}(\sigma_{32}) = -\frac{g_1 g_\mathrm{m}\left(g_1^2 - g_2^2\right)\left(g_0^2 - 2g_\mathrm{m}^2\right)}{2L}\cos\varPhi - \frac{g_\mathrm{m}^2 g_1^2 g_2}{L}\sin 2\varPhi, \tag{3b}$$

and the excited state population is given by

$$\rho_{33} = \frac{g_\mathrm{m}^2\left[\left(g_1^2 - g_2^2\right)^2 + 4g_1^2 g_2^2\sin^2\varPhi\right]}{L}, \tag{4}$$

with

$$L = \frac{1}{2}g_0^6 + g_\mathrm{m}^2\left[3\left(g_1^2 - g_2^2\right)^2 - 2g_0^4 + 2g_0^2 + 12g_1^2 g_2^2\sin^2\varPhi\right] + 2g_0^2 g_\mathrm{m}^4,$$
$$g_0^2 = g_1^2 + g_2^2,$$

and the relative phase $\varPhi$ is determined as

$$\varPhi = \left(\chi_1 - \chi_2\right) - \chi_\mathrm{m}. \tag{5}$$

One sees from equations (2–4), that the medium is absolutely transparent and not refractive for

$$\Phi = \pi n,\ n = 0, 1, 2, ... \tag{6}$$

and

$$g_1 = g_2. \tag{7}$$

These are exactly the conditions for the dark state in closed Λ system [14, 18–20].

For arbitrary optical field amplitudes and phases, however, the refraction and absorption (or amplification) of individual frequency components can be substantial. Here we are interested in generation of the optical field ω_2 with lowest possible losses of the e.m. energy. The total energy flow is proportional to intensity $I = I_1 + I_2$ of the optical waves. The intensity is expressed in terms of Rabi frequency as

$$I_n = (c/8\pi)\, E_n^2 = \left(2\hbar\omega_n^3/3\pi c^2\right) g_n^2 \gamma,$$

so that

$$\frac{\mathrm{d}I}{\mathrm{d}z} \sim \left(\frac{\mathrm{d}g_1^2}{\mathrm{d}\zeta} + \frac{\mathrm{d}g_2^2}{\mathrm{d}\zeta}\right) = -2\left(g_1 \mathrm{Im}\left(\sigma_{31}\right) + g_2 \mathrm{Im}\left(\sigma_{32}\right)\right) = -2\rho_{33},$$

where the last equality follows from the steady-state density matrix equations [14]. Thus, we arrive at almost obvious conclusion that dissipation of the e.m. energy is small when the excited state population is small: $\rho_{33} \ll 1$. Analysis of the expression (4) shows that, for arbitrary g_1, g_2 and Φ, this is the case for two ranges of the m.w. Rabi frequency: $g_{\mathrm{m}} \ll 1$, g_0 and $g_{\mathrm{m}} \gg 1$, g_0. These values correspond to EIT of the CPT-type and the Autler-Townes-type, respectively.

The change of the fields can be calculated analytically in present situation [21]. An interesting feature of the resonant case is that the phase equation can be solved for arbitrary values of g_{m}. The propagation equation for the relative phase is as follows (if we neglect the change of the m.w. field phase φ_{m} and the atomic dipole phases ϑ_l along the propagation path):

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta} = -\left(\frac{1}{g_1}\mathrm{Re}(\sigma_{31}) - \frac{1}{g_2}\mathrm{Re}(\sigma_{32})\right).$$

One can obtain from equations (2, 3) that

$$\frac{1}{g_1}\mathrm{Re}(\sigma_{31}) - \frac{1}{g_2}\mathrm{Re}(\sigma_{32}) = \frac{\cos\Phi}{\sin\Phi}\frac{g_2\mathrm{Im}(\sigma_{31}) + g_1\mathrm{Im}(\sigma_{32})}{g_1 g_2}$$

so that

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\zeta} = -\frac{\cos\Phi}{\sin\Phi}\frac{1}{g_1 g_2}\left(g_2\mathrm{Im}(\sigma_{31}) + g_1\mathrm{Im}(\sigma_{32})\right) = \frac{\cos\Phi}{\sin\Phi}\frac{1}{g_1 g_2}\frac{\mathrm{d}\left(g_1 g_2\right)}{\mathrm{d}\zeta}, \tag{8}$$

which can be integrated to give the constant of motion:

$$g_1 g_2 \cos\Phi = \Pi. \tag{9}$$

The constant Π is determined from the boundary conditions at the $\zeta = 0$. In particular, when one optical field is generated, $g_2(\zeta = 0) = 0$, we have constant value of $\cos\Phi$:

$$\cos\Phi(\zeta) = 0. \tag{10}$$

3 Frequency conversion assisted by the dark-state EIT

We now consider both EIT cases separately. For a weak m.w. field, $g_{\mathrm{m}} \ll 1$, the density matrix elements to the second order in g_{m} are:

$$\mathrm{Im}(\sigma_{31}) = -\frac{g_2 g_{\mathrm{m}}}{g_0^2}\sin\Phi + \frac{2g_{\mathrm{m}}^2 g_1\left(g_1^2 - g_2^2 + 2g_2^2\sin^2\Phi\right)}{g_0^6}, \tag{11a}$$

$$\mathrm{Im}(\sigma_{32}) = \frac{g_1 g_{\mathrm{m}}}{g_0^2}\sin\Phi - \frac{2g_{\mathrm{m}}^2 g_2\left(g_1^2 - g_2^2 - 2g_1^2\sin^2\Phi\right)}{g_0^6}. \tag{11b}$$

For the case of the field ω_2 generation, the dissipation of total optical energy is proportional to (taking into account Eq. (10)):

$$\frac{\mathrm{d}g_0^2}{\mathrm{d}\zeta} = -\frac{4g_{\mathrm{m}}^2}{g_0^2},$$

which has a solution

$$g_0^4 = g_0^4(\zeta = 0) - 8g_{\mathrm{m}}^2\zeta. \tag{12}$$

If we neglect this slow decay (which would simply correspond to neglect of terms of the second order in g_{m}), we obtain the following amplitude equations

$$\frac{\mathrm{d}g_1}{\mathrm{d}\zeta} = -\frac{g_2 g_m}{g_0^2}$$
$$g_2^2 = g_0^2 - g_1^2$$

which can be easily solved:

$$g_1^2 = g_0^2\cos^2\left(\frac{g_{\mathrm{m}}}{g_0^2}\zeta\right), \tag{13a}$$

$$g_2^2 = g_0^2\sin^2\left(\frac{g_{\mathrm{m}}}{g_0^2}\zeta\right). \tag{13b}$$

The solution indicates that the e.m. energy is transferred back and forth between two optical waves as the optical length increases. The period of these oscillations is $\zeta_\pi = \pi g_0^2/g_{\mathrm{m}}$, which is much smaller than the characteristic length of the total energy dissipation:

$$\zeta_{\mathrm{diss}} \approx g_0^2(\zeta = 0)/4g_{\mathrm{m}}^2,$$

cf. equation (12). Therefore, very efficient conversion takes place at

$$\zeta_{\max} = \pi g_0^2(\zeta = 0)/2g_{\mathrm{m}}. \tag{14}$$

The loss of the optical intensity is $\Delta I / I = 2\pi g_{\mathrm{m}} \ll 1$ at this point.

The reason for such an efficient process is a preparation of the medium in almost dark state. If the e.m. field between the states $|1\rangle$ and $|2\rangle$ is not applied then the dark state in Λ system is [9]:

$$|D\rangle = \frac{g_2}{g_0}|1\rangle - \exp(\mathrm{i}\Phi)\frac{g_1}{g_0}|2\rangle . \tag{15}$$

Therefore, the population of the dark state can be expressed in terms of the ground-state density matrix elements:

$$\rho_{\mathrm{DD}} = \frac{g_2^2}{g_0^2}\rho_{11} + \frac{g_1^2}{g_0^2}\rho_{22} - \frac{2g_1g_2}{g_0^2}\mathrm{Re}\left(\sigma_{21}\exp(-\mathrm{i}\Phi)\right),$$

which are, to the first order in g_{m},

$$\begin{aligned}
\mathrm{Im}(\sigma_{21}) &= \frac{g_1g_2}{g_0^2}\sin\Phi - \frac{g_{\mathrm{m}}\left(g_1^2 - g_2^2\right)}{g_0^4},\\
\mathrm{Re}(\sigma_{21}) &= -\frac{g_1g_2}{g_0^2}\cos\Phi,\\
\rho_{11} &= \frac{g_2^2}{g_0^2} - \frac{2g_{\mathrm{m}}g_1g_2}{g_0^4}\sin\Phi,\\
\rho_{22} &= \frac{g_1^2}{g_0^2} + \frac{2g_{\mathrm{m}}g_1g_2}{g_0^4}\sin\Phi.
\end{aligned}$$

Thus, $\rho_{\mathrm{DD}} = 1 - O\left(g_{\mathrm{m}}^2\right)$, which means that atoms all over the medium are in (almost) dark state, despite of the spatial changes of individual optical intensities. It is interesting that a large lower-level coherence is not established in advance (since $g_2(\zeta = 0) = 0$). However, as soon as ω_2 wave is generated, the coherence emerges, and the medium is prepared in the nonabsorbing state.

Even in real situation, when both the relaxation rate Γ of the coherence between states $|1\rangle$ and $|2\rangle$ and the Doppler broadening are present, the parameters of the process are in fairly good agreement with analytical results. Below we present numerical calculations assuming that the medium is a vapor of ^{23}Na atoms interacting with radiation in a Λ configuration of levels

$$3^2\mathrm{S}_{1/2}(F = 1)\text{–}3^2\mathrm{S}_{1/2}(F = 2)\text{–}3^2\mathrm{P}_{1/2}(F' = 2).$$

When the polarization of all e.m. waves is linear (say, along the y-axis) and external magnetic field is zero, there are two identical Λ systems of Zeeman sublevels with magnetic numbers $m_F = -1$ and $m_F = +1$. In the absence of m.w. field the sublevel $F = 2, m_F = 0$ is not coupled by radiation. Hence, a large part of atomic population can be optically pumped into the $F = 2, m_F = 0$ state and switched off the interaction. This problem is naturally resolved when the m.w. radiation is present: the population getting into $F = 2, m_F = 0$ is transferred by m.w. field to the state $F = 1, m_F = 0$ from where it is pumped back into Λ systems. Therefore, in numerical simulations we don't take into account the whole hyperfine structure and consider atoms as single Λ systems.

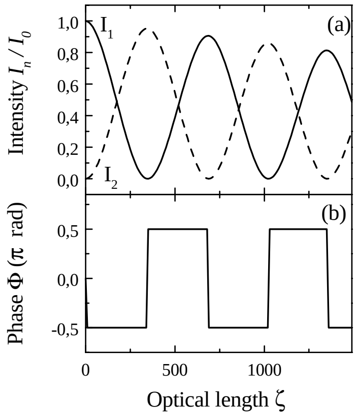

Fig. 2. Spatial variations of: (a) optical field intensities I_1 (solid curve) and I_2 (dashed curve) in units of input intensity $I_0 \equiv I_1(\zeta = 0)$, (b) the relative phase Φ, in a vapor of ^{23}Na atoms interacting with radiation in a Λ configuration of levels $3^2\mathrm{S}_{1/2}(F = 1)$–$3\,^2\mathrm{S}_{1/2}(F = 2)$–$3^2\mathrm{P}_{1/2}$. Vapor temperature $T = 440$ K, $\Gamma = 10^{-4}\gamma$, detunings $\Delta_1 = \Delta_2 = 0$, Rabi frequencies of input fields $g_1(\zeta = 0) = 2.0$, $g_{\mathrm{m}} = 0.02$.

In Figure 2 the spatial dependence of the field intensities and the phase Φ are plotted for Na temperature $T = 440$ K (this gives the saturated vapor density of $N = 4.42 \times 10^{11}$ cm^{-3} and corresponds to a most probable velocity of atoms of $v_{\mathrm{p}} = 5.64 \times 10^4$ cm/s), $\Gamma = 10^{-4}\gamma$ (1 kHz), input Rabi frequencies $g_1(\zeta = 0) = 2.0$, $g_{\mathrm{m}} = 0.02$ (corresponding to intensities of $I_1 = 12.6$ mW/cm^2 and $I_{\mathrm{m}} = 1.26\,\mu$W/cm^2). We see that dynamics of the intensities and the phase does not change qualitatively as compared to the case of negligible decay of the dark state. The behavior of the phase Φ in Figure 2b follows the law $\cos\Phi(\zeta) = 0$ obtained analytically. The jumps in the phase occur at points where the intensity of the field being absorbed approaches zero, according to equation (1b). The ω_2 wave is generated and reaches its maximum at the length $\zeta = 340$ (this corresponds to the real length of the gas cell of $z = 1.9$ cm). This value is quite close to that calculated from analytical results, equation (14), $\zeta_{\max} = 314$. The maximum intensity of the ω_2 wave is $I_2/I_1(\zeta = 0) = 0.952$ which is slightly below the value 0.966 calculated from equation (12) because of an additional dissipation due to decay of the dark state with the rate Γ . Obviously, this rate must be sufficiently small in order to allow for the population trapping in $|D\rangle$, namely it must be much smaller than the optical pumping rate into the dark state:

$$\Gamma/\gamma \ll \frac{g_0^2}{1 + \Delta^2} \, . \tag{16}$$

Here, detuning Δ includes the Doppler shift:

$$\gamma\Delta = \Delta_1 - k_1v_z \approx \Delta_2 - k_2v_z,$$

where $\Delta_n = \omega_n - (\mathcal{E}_3 - \mathcal{E}_n)/\hbar$ are the laser frequency detunings from transitions $|n\rangle$–$|3\rangle$, $(n = 1, 2)$, $\mathcal{E}_n$ is the eigenenergy of the atomic state $|n\rangle$. For the resonance $\Delta_1 = \Delta_2 = 0$ and large Doppler broadening $k_1v_{\mathrm{p}} \gg \gamma$,

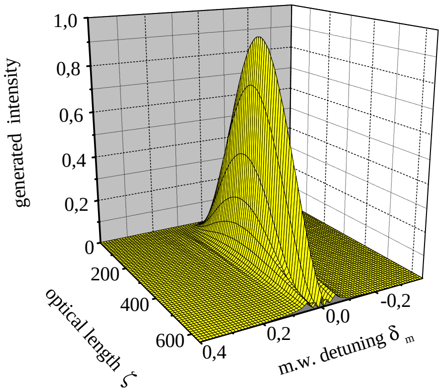

Fig. 3. Generated intensity I_2/I_0 as a function of the optical length ζ and the microwave detuning $\delta_{\mathrm{m}} = \omega_{\mathrm{m}} - (\mathcal{E}_2 - \mathcal{E}_1)/\hbar$ (in units of the excited state relaxation rate γ). Other parameters are the same as in Figure 2.

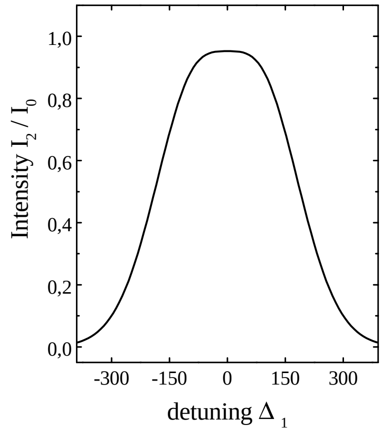

Fig. 4. Dependence of the generated intensity I_2/I_0 on detuning Δ_1 (in units of the excited state relaxation rate γ) for fixed $\omega_{\mathrm{m}} = (\mathcal{E}_2 - \mathcal{E}_1)/\hbar$, at the optical length $\zeta = 340$. Other parameters are the same as in Figure 2.

condition (16) reduces to

$$g_0^2 \gg \frac{\Gamma}{\gamma}\left(\frac{k_1 v_{\mathrm{p}}}{\gamma}\right)^2 . \tag{17}$$

The better this condition is satisfied, the higher the efficiency of frequency conversion is. The rate Γ is determined by m.w. field fluctuations, atomic collisions and other random phase disturbing processes. For parameters of the proposed here experiment, Γ can be very small. Recently, the rate $\Gamma < 50$ Hz has been observed in experiment [22].

Inasmuch as CPT is a basis for the considered above scheme, the generation occurs under quite restrictive conditions on e.m. wave frequencies. It is well known that CPT takes place when the optical frequencies are in the narrow range ("black line") around two-photon resonance [9]:

$$\Delta_2 = \Delta_1. \tag{18}$$

Considering that the wave E_2 is always generated at frequency $\omega_2 = \omega_1 - \omega_{\mathrm{m}}$ (simply due to photon energy conservation), the condition (18) tells us that the generation takes place only if the m.w. field frequency is in the narrow range around resonance with transition $|1\rangle$–$|2\rangle$: $\omega_{\mathrm{m}} = (\mathcal{E}_2 - \mathcal{E}_1)/\hbar$. Figure 3 demonstrates this fact. The width of the generation peak (width of the black line) is determined by the pumping rate into the dark state [9]. In the presence of large Doppler broadening this width is of the order of $\delta\omega_{\mathrm{m}} \approx \gamma g_0^2/(k_1 v_{\mathrm{p}}/\gamma)^2$, which is a few kHz for parameters of Figure 3.

It is interesting that, at the same time, the large Doppler broadening allows for a broad tuning of the generated wave. In Figure 4 we have plotted dependence of the generated intensity on detuning Δ_1 (for fixed $\omega_{\mathrm{m}} = (\mathcal{E}_2 - \mathcal{E}_1)/\hbar$) at the optical length $\zeta = 340$. One can see that conversion efficiency remains fairly large for detunings of the order of the Doppler broadening (few GHz for Na vapor). This is because the CPT survives even at large common detunings $\Delta_2 = \Delta_1$ as long as the condition (16) is satisfied.

4 Frequency conversion assisted by the Autler-Townes-type EIT

The second type of EIT allowing efficient frequency conversion in the Λ medium takes place at strong m.w. fields, $g_{\mathrm{m}} \gg 1, g_0$. In this case, the absorption coefficients to the second order in $(1/g_{\mathrm{m}})$ are:

$$\mathrm{Im}(\sigma_{31}) = \frac{g_2}{2g_{\mathrm{m}}}\sin\Phi + \frac{g_1\left(g_1^2 - g_2^2 + 2g_2^2\sin^2\Phi\right)}{2g_{\mathrm{m}}^2 g_0^2}, \tag{19a}$$

$$\mathrm{Im}(\sigma_{32}) = -\frac{g_1}{2g_{\mathrm{m}}}\sin\Phi - \frac{g_2\left(g_1^2 - g_2^2 - 2g_1^2\sin^2\Phi\right)}{2g_{\mathrm{m}}^2 g_0^2}. \tag{19b}$$

The energy dissipation is determined by the equation

$$\frac{\mathrm{d}g_0^2}{\mathrm{d}\zeta} = -\frac{g_0^2}{g_{\mathrm{m}}^2},$$

with a solution

$$g_0^2 = g_0^2(\zeta = 0)\exp\left(-g_{\mathrm{m}}^{-2}\zeta\right). \tag{20}$$

Again, if we neglect the slow total energy dissipation (*i.e.*, we neglect terms to the second order in $1/g_{\mathrm{m}}$ in Eq. (19a)), we obtain the solution of amplitude equations, very similar to the CPT case:

$$g_1^2 = g_0^2\cos^2\left(\frac{1}{2g_{\mathrm{m}}}\zeta\right), \tag{21a}$$

$$g_2^2 = g_0^2\sin^2\left(\frac{1}{2g_{\mathrm{m}}}\zeta\right). \tag{21b}$$

Here, the period of intensity oscillations is $\zeta_\pi = 2\pi g_{\mathrm{m}}$, which is again much smaller than the characteristic length of the total energy dissipation: $\zeta_{\mathrm{diss}} \approx g_{\mathrm{m}}^2$. Maximum energy transfer to the ω_2 field occurs at

$$\zeta_{\max} = \pi g_{\mathrm{m}}. \tag{22}$$

The loss of the optical intensity is

$$\Delta I \,/\, I = 1 - \exp\left(-\pi/g_{\mathrm{m}}\right) \ll 1$$

at this point.

Numerical calculations of the optical waves propagation give the results which are in very good agreement with analytical ones, and which are qualitatively very similar to the CPT-case in Figure 2. The physical mechanism is, however, different. The CPT effect does not work at strong m.w. fields except under the specific conditions (6, 7). This can be proved by considering the density matrix elements. It turns out that for this case the ground state populations are equal to $\rho_{11} = \rho_{22} = 0.5$ up to the second order in $(1/g_{\mathrm{m}})$,

$$\mathrm{Im}(\sigma_{21}) = O\left(1/g_{\mathrm{m}}^2\right)$$

and

$$\mathrm{Re}(\sigma_{21}) = -\left(g_1 g_2/g_0^2\right)\cos\Phi + O\left(1/g_{\mathrm{m}}^2\right).$$

Therefore, the atomic population is, for arbitrary g_1, g_2 and Φ, not all pumped into the dark state:

$$\rho_{\mathrm{DD}} = 1/2 + \left(2g_1^2 g_2^2/g_0^4\right)\cos^2\Phi < 1.$$

Under the condition (10) taking place at the generation, the ground-state coherence σ_{21} is negligibly small and $\rho_{\mathrm{DD}} = 1/2$.

The strong reduction of optical absorption in present case may be explained, for example, in a following way. One can view the ground states $|1\rangle$ and $|2\rangle$ excited by m.w. field as two dressed states:

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle \pm \exp\left(\mathrm{i}\chi_{\mathrm{m}}\right)|2\rangle\right), \tag{23}$$

separated by $2g_{\mathrm{m}}$ through the AC-Stark splitting or Autler-Townes effect [10]. Total absorption of weak optical waves ω_1 and ω_2 can be monitored by the population of excited state $|3\rangle$ (Fig. 5). The absorption is maximum when the optical fields are tuned to resonance with transition from state $|3\rangle$ to either $|+\rangle$ or $|-\rangle$. When the optical fields are tuned to the middle of the two dressed states (*i.e.* to the resonance with corresponding transitions $|3\rangle$–$|1\rangle$ and $|3\rangle$–$|2\rangle$), two effects contribute to the value of absorption. One is the AC-Stark splitting, which reduces absorption, and the other is the interference between the two transition paths of $|3\rangle$–$|+\rangle$ and $|3\rangle$–$|-\rangle$ made by both optical fields. The interference is destructive under the conditions (6, 7). Note that in this case either $|-\rangle$ or $|+\rangle$, depending on whether Φ is 0 or π, coincides with the dark state $|D\rangle$, equation (15). However, the generation implies $\cos\Phi = 0$, producing enhanced absorption by constructive interference. Nevertheless, the excited state population ρ_{33} remains on a level of $g_0^2/g_{\mathrm{m}}^2 \ll 1$ (*cf.* Eq. (4)), given by the wings of the dressed state absorption peaks.

Thus, a very weak optical absorption at strong m.w. field is only due to the Autler-Townes effect. This mechanism requires large m.w. intensities, but it has some advantages over the CPT-case. First of all, it is more robust: the relaxation Γ does not play so important role,

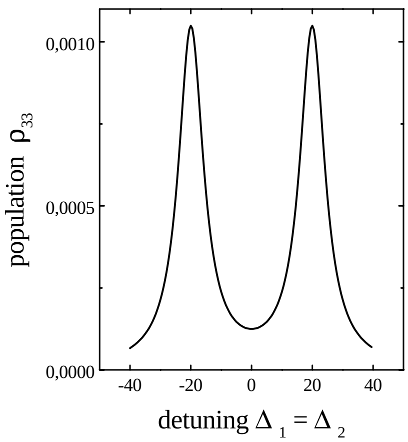

Fig. 5. Population of the excited state $|3\rangle$ as a function of common optical detuning $\Delta_1 = \Delta_2$ for fixed $\omega_{\mathrm{m}} = (\mathcal{E}_2 - \mathcal{E}_1)/\hbar$. Rabi frequencies are $g_1 = 0.3$, $g_2 = 0.1$, $g_{\mathrm{m}} = 20.0$, relative phase $\Phi = \pi/2$, $\Gamma = 10^{-3}\gamma$.

and the range of m.w. frequency ω_{m} where generation occurs is much broader (it is of order of g_{m}) as compared to the case of weak m.w. field. Similar to the case with a weak m.w. field, there is a possibility to tune the generated radiation over the Doppler contour, and here the tuning is not as sensitive to the value of Γ as in former case. Note that here the optical length scales are determined only by g_{m} and do not depend on the input intensity, $g_0^2(\zeta = 0)$. Therefore, the conversion is maximum at the same length for different input intensities. This is especially important for experiments with nonuniform light beams, *e.g.*, with the Gaussian intensity profile. Another important advantage is that this mechanism can be applied not only to Λ-system, but also to a V-scheme with one ground and two excited states coupled by m.w. field. We believe that the optical frequency conversion may be observed experimentally, for example, in a V-system of Pr^{3+}: $YAlO_3$ solid where the reduced absorption was recently demonstrated [13].

Finally, Figure 6 represents the dependence of generated wave on Rabi frequency g_{m} of the m.w. field at fixed optical length $\zeta = 340$. This figure clearly demonstrates two ranges of g_{m} where EIT and, correspondingly, efficient frequency conversion take place.

5 Conclusion

We have described a scheme for efficient optical conversion based on EIT in atomic Λ system where the interaction loop is closed by a microwave field. Depending on the m.w. intensity, two mechanisms of EIT work in this scheme: CPT and Autler-Townes effects. Intensity of the m.w. field plays also a role of controlling parameter in the conversion process – it determines optical length scales of the process, *cf.* equations (14, 22), as well as the degree of the total energy dissipation, *cf.* equations (12, 20). An optimal choice is always possible, which should allow an experimental realization of the proposed scheme

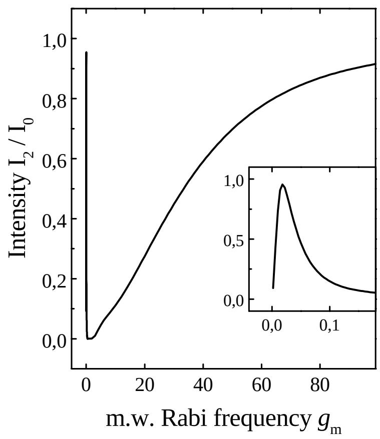

Fig. 6. Dependence of the generated intensity I_2/I_0 on the Rabi frequency of microwave field g_m at the optical length $\zeta = 340$. Other parameters are the same as in Figure 2. Inset shows the range of small g_m.

in different systems. Since the EIT-assisted frequency conversion combines large nonlinearity with substantially reduced spontaneous emission noise, one may expect that the generated signal will be fluctuation-correlated with the pump wave [23]. Such correlations persist even on a quantum level [24]. Therefore, the present scheme can be used for generation of two phase-correlated optical waves, which would be an alternative to conventional methods using electro- or acousto-optical modulators, or direct current modulation in laser diodes. Other possible applications may be optical phase conjugation [25] and generation of squeezed light [26].

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References

1. R.W. Boyd, *Nonlinear Optics* (Academic Press, San Diego, 1992).
2. S.E. Harris, J.E. Feld, A. Imamoglu, Phys. Rev. Lett. **64**, 1107 (1990).
3. S.E. Harris, Phys. Today **50**, 36 (1997).
4. K. Hakuta, L. Marmet, B.P. Stoicheff, Phys. Rev. Lett. **66**, 596 (1991).
5. S. Babin, U. Hinze, E. Tiemann, B. Wellegehausen, Opt. Lett. **21**, 1186 (1996); A. Apolonskii, S. Baluschev, U. Hinze, E. Tiemann, B. Wellegehausen, Appl. Phys. B **64**, 435 (1997).
6. B.S. Ham, M.S. Shahriar, P.R. Hemmer, Opt. Lett. **22**, 1138 (1997); Opt. Lett. **24**, 86 (1999).
7. M. Jain, H. Xia, G.Y. Yin, A.J. Merriam, S.E. Harris, Phys. Rev. Lett. **77**, 4326 (1996).
8. A.J. Merriam, S.J. Sharpe, H. Xia, D. Manuszak, G.Y. Yin, S.E. Harris, Opt. Lett. **24**, 625 (1999).
9. E. Arimondo, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1996), Vol. 35, p. 257.
10. C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).
11. Y. Zhao, C. Wu, B.-S. Ham, M.K. Kim, E. Awad, Phys. Rev. Lett. **79**, 641 (1997).
12. B.S. Ham, M.S. Shahriar, P.R. Hemmer, J. Opt. Soc. Am. B **15**, 1541 (1998).
13. K. Yamamoto, K. Ichimura, N. Gemma, Phys. Rev. A **58**, 2460 (1998).
14. D.V. Kosachiov, Kvant. Elektron. **22**, 1123 (1995) [Quant. Electronics **25**, 1089 (1995)].
15. E.A. Korsunsky, D.V. Kosachiov, Phys. Rev. A **60**, 4996 (1999).
16. A. Godone, F. Levy, J. Vanier, Phys. Rev. A **59**, R12 (1999); J. Vanier, A. Godone, F. Levy, Phys. Rev. A **58**, 2345 (1998).
17. E.A. Korsunsky, D.V. Kosachiov, J. Opt. Soc. Am. B **17** (in press, 2000).
18. D.V. Kosachiov, B.G. Matisov, Yu.V. Rozhdestvensky, J. Phys. B **25**, 2473 (1992).
19. S.J. Buckle, S.M. Barnett, P.L. Knight, M.A. Lauder, D.T. Pegg, Opt. Acta **33**, 2473 (1986).
20. D. Kosachiov, B. Matisov, Yu. Rozhdestvensky, Opt. Commun. **85**, 209 (1991).
21. J.A. Armstrong, N. Bloembergen, J. Ducuing, P.S. Pershan, Phys. Rev. **127**, 1918 (1962).
22. S. Brandt, A. Nagel, R. Wynands, D. Meschede, Phys. Rev. A **56**, R1063 (1997).
23. T.T. Grove, M.S. Shahriar, P.R. Hemmer, P. Kumar, V.S. Sudarshanam, M. Cronin-Golomb, Opt. Lett. **22**, 769 (1997).
24. M.D. Lukin, A.B. Matsko, M. Fleischhauer, M.O. Scully, Phys. Rev. Lett. **82**, 1847 (1999).
25. P.R. Hemmer, D.P. Katz, J. Donoghue, M. Cronin-Golomb, M.S. Shahriar, P. Kumar, Opt. Lett. **20**, 982 (1995); V.S. Sudarshanam, M. Cronin-Golomb, P.R. Hemmer, M.S. Shahriar, Opt. Lett. **22**, 1141 (1997); T.T. Grove, E. Rousseau, Xiao-Wei Xia, D.S. Hsiung, M.S. Shahriar, P.R. Hemmer, Opt. Lett. **22**, 1677 (1997).
26. M.I. Kolobov, P. Kumar, Opt. Lett. **18**, 849 (1993); P. Kumar, M.I. Kolobov, Opt. Commun. **104**, 374 (1994).